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The Bogomol'nyi Bound of Lee-Weinberg Magnetic Monopoles

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Abstract

The Lee-Weinberg $U(1)$ magnetic monopoles, which have been reinterpreted as topological solitons of a certain non-Abelian gauged Higgs model recently, are considered for some specific choice of Higgs couplings. The model under consideration is shown to admit a Bogomol'nyi-type bound which is saturated by the configurations satisfying the generalized BPS equations. We consider the spherically symmetric monopole solutions in some detail.

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Recently, Lee and Weinberg [1] constructed a new class of finite-energy magnetic monopoles in the context of a purely Abelian gauge theory. The corresponding $U(1)$ gauge potential is simply that of a point-like Dirac monopole [2] (with the monopole strength satisfying the Dirac quantization condition), yet the total energy is rendered finite by introducing a charged vector field of arbitrary positive gyromagnetic ratio and by fine-tuning a quartic self-interaction. This is in a marked contrast to finite-mass magnetic monopoles of the 't Hooft-Polyakov type [3], which appear as topological solitons of some spontaneously broken non-Abelian gauge theories. In the latter case, the existence of such solitons and the charge quantization [2] thereof are understood in terms of the nontrivial second homotopy group of the appropriate vacuum manifold.

In Ref. [4], however, we realized that the above Lee-Weinberg monopole is also equipped with a hidden, spontaneously broken $SO(3)$ gauge symmetry. This naturally explains the integer-charged Lee-Weinberg monopoles as topological solitons associated with the vacuum manifold $SO(3)/U(1)$, just as in the usual $SO(3)$ Higgs model. In the present paper, we will determine the *self-dual* limit of the Lee-Weinberg model, in which the energy functional satisfies a simple Bogomol'nyi-type bound. This is achieved for a special form of the Higgs potential, but the gyromagnetic ratio can still assume an arbitrary positive value. [For $g = 2$, our model reduces to the old Bogomol'nyi-Prasad-Sommerfield (BPS) model [5].] Configurations that saturate the thus-obtained Bogomol'nyi bound solve certain first-order differential equations; they generalize the old BPS equations, the investigation of which has been an important part of mathematical physics for the last two decades [6]. These generalized BPS monopoles satisfy the same mass formula as the old BPS monopoles. Also given is a simple argument which demonstrates the existence of unit-charged monopole solutions to our generalized BPS equations, while satisfying the physical boundary conditions. There is a strong evidence that static multi-monopole solutions exist in this model as well.

Let us recapitulate the observations of Refs. [1] and [4] first. The Abelian model of Ref. [1] consists of a $U(1)$ electromagnetic potential A_μ, a charged vector field W_μ, and a real scalar ϕ, with the Lagrangian density chosen to have the form

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}|\bar{D}_\mu W_\nu - \bar{D}_\nu W_\mu|^2 + \frac{g}{4}H_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}H_{\mu\nu}H^{\mu\nu}$$
$$-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - m^2(\phi)|W_\mu|^2 - V(\phi)\,, \tag{1}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $\bar{D}_\mu W_\nu \equiv \partial_\mu W_\nu + ieA_\mu W_\nu$, and $H_{\mu\nu} \equiv ie(W^*_\mu W_\nu - W^*_\nu W_\mu)$. The coupling g is the gyromagnetic ratio associated with the magnetic moment of the charged vector, and $m^2(\phi)$ is assumed to vanish at $\phi = 0$ but is equal to $m_W^2(\neq 0)$ when ϕ is at its (nontrivial) vacuum value. With $g = 2$, $\lambda = 1$ and $m(\phi) = e\phi$, this is nothing but the unitary gauge version of the spontaneously broken non-Abelian gauge theory of Ref. [3] and thus renormalizable; but for generic values of g and λ, the theory is nonrenormalizable. Author of Ref. [1] noted that if the couplings satisfy the relation $\lambda = \frac{g^2}{4}$ (for arbitrary positive g), the model allows magnetic monopoles with *finite* mass.

The above model can be recast as a nonrenormalizable $SO(3)$ gauge theory with the gauge connection 1-form $B = (B^a_\mu dx^\mu)T^a$ and a triplet Higgs $\Phi = \Phi^a T^a$. Specifically, consider the theory defined by the Lagrangian density [4]

$$\mathcal{L}' = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{g-2}{4}\mathcal{F}_{\mu\nu}\mathcal{H}^{\mu\nu} - \frac{\lambda-1}{4}\mathcal{H}_{\mu\nu}\mathcal{H}^{\mu\nu} - \frac{1}{2}(D_\mu\Phi)^a(D^\mu\Phi)^a$$
$$-\frac{1}{e^2}(m^2(|\phi|) - e^2\Phi^a\Phi^a)(D_\mu\hat{\Phi})^a(D^\mu\hat{\Phi})^a - V(|\Phi|)\,, \tag{2}$$

where G is the non-Abelian field strength associated with B, $(D_\mu\Phi)^a \equiv \partial_\mu\Phi^a + e\epsilon^{abc}B^b_\mu\Phi^c$, $\hat{\Phi}^a \equiv \Phi^a/|\Phi|$, and we have defined two (gauge-invariant) tensors $\mathcal{F}_{\mu\nu}$, $\mathcal{H}_{\mu\nu}$ by

$$\mathcal{F}_{\mu\nu} - \mathcal{H}_{\mu\nu} = G^a_{\mu\nu}\Phi^a\,, \quad \mathcal{H}_{\mu\nu} = -\frac{1}{e}\epsilon_{abc}\hat{\Phi}^a(D_\mu\hat{\Phi})^b(D_\nu\hat{\Phi})^c\,. \tag{3}$$

Note that 't Hooft [3] previously used the tensor $\mathcal{F}$ to represent physical electromagnetic fields. Now, in the unitary gauge (i.e., $\hat{\Phi}^a = \delta^{a3}$), we may identify $|\Phi|$ with $|\phi|$, B^3_μ with A_μ, and $\frac{1}{\sqrt{2}}(B^1_\mu + iB^2_\mu)$ with W_μ; then, we have $\mathcal{F}_{\mu\nu} = F_{\mu\nu}$, $\mathcal{H}_{\mu\nu} = H_{\mu\nu}$, $(D^\mu\hat{\Phi})^a(D_\mu\hat{\Phi})^a = 2e^2W^*_\mu W^\mu$, etc. As a result, the $SO(3)$ invariant Lagrangian density $\mathcal{L}'$ reduces to the expression (1) of the apparently Abelian theory of Lee and Weinberg. This in turn allows us to reinterpret all integer-charged Lee-Weinberg monopoles as topological solitons of the non-Abelian theory defined by $\mathcal{L}'$. In the latter description we can have the monopoles

described in a non-singular (i.e., string-free) gauge, and if n is the winding number associated with the map $\hat{\Phi}^a(r=\infty) : S^2 \to S^2$, the magnetic monopole strength is simply given by $g_{\rm magnetic} = -\frac{4\pi n}{e}$ [7]. Especially, the unit-charged ($n = \pm 1$) Lee-Weinberg monopoles may be described by the familiar hedgehog form:

$$\Phi^a = \hat{x}^a \phi(r)\,, \qquad B^a = -\frac{\epsilon_{abc} x^b dx^c}{er^2}[1-u(r)]\,. \tag{4}$$

Aside from the topological argument, one generally needs to look into the energetics at short distances to ascertain the existence of actual finite-energy monopole solutions. As mentioned already, Lee and Weinberg showed that the monopoles have finite energy if $\lambda = g^2/4$ with arbitrary positive g; this conclusion was confirmed also [4] using the spherically symmetric form (4) in the equivalent non-Abelian description. But, $u^2(0) = \frac{2}{g}$ for these Lee-Weinberg monopoles and thus the corresponding vector potentials are not regular at the origin (except for the special case $g = 2$, corresponding to the renormalizable model). More detailed study on these solutions may be carried out with the help of the field equations (with $\frac{g^2}{4} = \lambda$). For the radial functions $\phi(r)$ and $u(r)$, they imply

$$\frac{d^2u}{dr^2} + \frac{g}{2}\left(1 - \frac{g}{2}u^2\right)\frac{u}{r^2} = m^2(\phi)u\,, \tag{5a}$$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{u^2}{e^2r^2}\frac{dm^2(\phi)}{d\phi} = \frac{dV(\phi)}{d\phi}\,. \tag{5b}$$

Making a local analysis with these equations near the origin, it is then easy to show that $u(r)$ has the following behavior near the origin:

$$\left[u^2(r) - \frac{2}{g}\right] \sim r^\alpha\,, \qquad \text{with } \alpha = \frac{1}{2} + \sqrt{g + \frac{1}{4}}\,. \tag{6}$$

Note that if the gyromagnetic ratio takes the special value $g = 2$, this reduces to the expected analytic behavior with $\alpha = 2$. The behavior (6) will be sufficient to remove possible boundary contribution at the origin, thus enabling us to find the Bogomol'nyi bound through the usual argument.

We may also recall the usual BPS limit of the renormalizable case, which is realized when the Higgs potential $V(\phi)$ is dropped and the couplings are such that $1 = \frac{g}{2} = \lambda =$

$m^2(\phi)/e^2\phi^2$. The energy functional for purely magnetic, static configurations can then be expressed as [5]

$$\begin{aligned}
\mathcal{E}_0 &= \int d^3x \left\{ \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{2} (D_i\Phi)^a (D_i\Phi)^a \right\} \\
&= \int d^3x \frac{1}{2} \left\{ (D_i\Phi)^a \mp \frac{1}{2}\epsilon_{ijk} G^a_{jk} \right\} \left\{ (D_i\Phi)^a \mp \frac{1}{2}\epsilon_{ijk} G^a_{jk} \right\} \\
&\quad \pm \int d^3x \frac{1}{2} \epsilon^{ijk} \partial_i (\Phi^a G^a_{jk}) \\
&\geq \left| \frac{1}{2} \oint_{r=\infty} dS_i \epsilon^{ijk} \Phi^a G^a_{jk} \right| ,
\end{aligned} \tag{7}$$

where we have used the non-Abelian Bianchi identity in performing the partial integration. The surface integral is equal to $-\frac{4\pi n}{e}\phi(\infty)$, and thus we are led to the Bogomol'nyi bound

$$\mathcal{E}_0 \geq \left| \frac{4\pi n}{e} \phi(\infty) \right| . \tag{8}$$

The so-called BPS monopoles (or self-dual monopoles) are configurations that saturate this energy bound. It follows from the bulk part of (7) that they must solve the BPS equations (or self-duality equations)

$$(D_i\Phi)^a = \pm \frac{1}{2} \epsilon_{ijk} G^a_{jk} , \tag{9}$$

where the sign is determined by that of n. Especially, if we insert the hedgehog ansatz (4) into (9), we obtain the first-order equations for the unit-charged BPS monopoles:

$$\frac{du}{dr} = \pm e\phi u , \qquad e\frac{d\phi}{dr} = \mp \frac{1}{r^2}(1-u^2) . \tag{10}$$

The solutions to these equations are given in terms of elementary functions. [See (21) below.]

Now the question is whether we can have a similar Bogomol'nyi limit for general values of the gyromagnetic ratio g within the model defined by the Lagrangian density (2). The first clue that this might be possible comes from studying the energy functional of the given model for spherically symmetric configurations (i.e., for the form (4)):

$$\mathcal{E} = \int d^3x \left\{ \frac{1}{e^2r^2} \left(\frac{du}{dr} \right)^2 + \frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + \frac{1}{2e^2r^4} [\lambda u^4 - gu^2 + 1] + \frac{u^2}{e^2r^2} m^2(\phi) + V(\phi) \right\} . \tag{11}$$

We again drop the Higgs potential $V(\phi)$, but keep the gyromagnetic ratio $g(>0)$ arbitrary such that $\frac{g^2}{4}=\lambda$ and $m^2(\phi)=\lambda e^2\phi^2$. Recall that $\frac{g^2}{4}=\lambda$ was necessary to ensure the finite total energy. Then, the energy functional in (11) may be rewritten as

$$\begin{aligned}\mathcal{E}=4\pi\int_0^\infty dr\left\{\left(\frac{1}{e}\frac{du}{dr}\mp\frac{g}{2}u\phi\right)^2+\frac{1}{2}\left(r\frac{d\phi}{dr}\pm\frac{1}{er}\left(1-\frac{g}{2}u^2\right)\right)^2\right\}\\ \mp\frac{4\pi}{e}\int_0^\infty dr\frac{d}{dr}\left[\phi(1-\frac{g}{2}u^2)\right].\end{aligned}\tag{12}$$

As long as the combination $\phi(1-\frac{g}{2}u^2)$ vanishes at the origin (which is true for our case), $\mathcal{E}$ is manifestly bounded below by the value $|4\pi\phi(\infty)/e|$. The generalized first-order equations, which can be read off from the bulk part of (12), are

$$\frac{du}{dr}=\pm\frac{g}{2}e\phi u\,,\quad e\frac{d\phi}{dr}=\mp\frac{1}{r^2}\left(1-\frac{g}{2}u^2\right).\tag{13}$$

It is comforting to see that, in the renormalizable limit $\frac{g}{2}\to 1$, these reduce to (10).

We will now demonstrate that once we choose $V(\phi)=0$, $g^2/4=\lambda$ and $m^2(\phi)=\lambda e^2\phi^2$ with the Lagrangian density (2), a generalized Bogomol'nyi system results without the assumption of the spherical symmetry. The static energy functional for purely magnetic configurations (i.e., with $B_0^a\equiv 0$) reads

$$\begin{aligned}\mathcal{E}=\int d^3x\left\{\frac{1}{4}\left(G^a_{ij}-(\frac{g}{2}-1)\hat{\Phi}^a\mathcal{H}_{ij}\right)\left(G^a_{ij}-(\frac{g}{2}-1)\hat{\Phi}^a\mathcal{H}_{ij}\right)\right.\\ \left.+\frac{1}{2}\left((D_i\Phi)^a+(\frac{g}{2}-1)\phi(D_i\hat{\Phi})^a\right)\left((D_i\Phi)^a+(\frac{g}{2}-1)\phi(D_i\hat{\Phi})^a\right)\right\}.\end{aligned}\tag{14}$$

Then, it is a matter of straightforward algebra using the non-Abelian Bianchi identity and the relation $\hat{\Phi}^aG^a_{\mu\nu}\equiv\mathcal{F}_{\mu\nu}-\mathcal{H}_{\mu\nu}$ to rewrite this in a form analogous to (7),

$$\begin{aligned}\mathcal{E}=\int d^3x\frac{1}{2}\left\{(D_i\Phi)^a+(\frac{g}{2}-1)\phi(D_i\hat{\Phi})^a\mp\frac{1}{2}\epsilon_{ijk}\left(G^a_{jk}-(\frac{g}{2}-1)\hat{\Phi}^a\mathcal{H}_{jk}\right)\right\}^2\\ \pm\int d^3x\frac{1}{2}\epsilon^{ijk}\partial_i\left\{\phi\mathcal{F}_{jk}-\frac{g}{2}\phi\mathcal{H}_{jk}\right\}\pm\int d^3x(\frac{g}{2}-1)\phi[\epsilon^{ijk}\partial_i\mathcal{F}_{jk}]\,.\end{aligned}\tag{15}$$

The 't Hooft tensor $\mathcal{F}$ satisfies its own Bianchi identity $\epsilon^{ijk}\partial_i\mathcal{F}_{jk}\equiv 0$ whenever $\phi\neq 0$. Therefore, the last term on the right hand side of (15) gives a null contribution, while the second term may be changed to a surface integral at spatial infinity (at least for $g>0$).

Since $\mathcal{H}_{jk}$ approaches zero at spatial infinity faster than $1/r^2$, the surface integral is again given by $-4\pi n\phi(\infty)/e$. Thus, from (15), we have the energy bound which is independent of g:

$$\mathcal{E} \geq \left| \frac{4\pi n}{e} \phi(\infty) \right| , \quad \text{for all } g > 0 \, . \tag{16}$$

On the other hand, the minimum energy configurations for any given winding number n correspond to the solutions of the generalized BPS equations which have an explicit g-dependence:

$$(D_i \Phi)^a + (\frac{g}{2} - 1)\phi(D_i \hat{\Phi})^a = \pm \frac{1}{2} \epsilon_{ijk} \left(G^a_{jk} - (\frac{g}{2} - 1)\hat{\Phi}^a \mathcal{H}_{jk} \right) . \tag{17}$$

As one can easily demonstrate, solutions of the above generalized BPS equations automatically solve the full field equations. Also, if we use the hedgehog ansatz with these equations, we easily recover (13). Note that, in the unitary gauge, we can express these BPS equations by the following first-order differential equations:

$$\begin{aligned} \partial_i \phi &= \pm \epsilon_{ijk} \left(\partial_j A_k - i \frac{ge}{2} W^*_j W_k \right) , \\ \frac{g}{2} \phi W_i &= \pm \epsilon_{ijk} \bar{D}_j W_k \, . \end{aligned} \tag{18}$$

Solutions to these equations will provide us with all minimal energy magnetic monopole solutions (for any give n) in the special case of the Lee-Weinberg model, which is described by the Lagrangian density (1) with $V(\phi) = 0$, $g^2/4 = \lambda$ and $m^2(\phi) = \lambda e^2 \phi^2$. But, even for $n = 1$, no simple closed-form solution is known to us yet.

Still, some comments on the unit-charged BPS solutions for arbitrary $g(> 0)$ may be desirable. Note that (13) is invariant under a simultaneous rescaling of ϕ and r, thanks to the vanishing Higgs potential. The resulting one-parameter family of solutions are characterized by the Higgs expectation value $\phi(\infty)$, as one would expect. Defining $x \equiv |eg\phi(\infty)|r/2$, $K \equiv \sqrt{\frac{g}{2}} u$ and $h \equiv \phi/\phi(\infty)$, we can rewrite (13) as

$$\frac{dK}{dx} = -hK \, , \quad \frac{2}{g}\frac{dh}{dx} = \frac{1 - K^2}{x^2} \, . \tag{19}$$

For a finite-energy solution, we should then require that $K(\infty) = 0$ and $h(\infty) = 1$. Also, near the origin, the expected behaviors for $K(x)$ and $h(x)$ are

$$K(x) = 1 - K_1 x^\alpha + \cdots\,, \quad xh(x) = \alpha K_1 x^\alpha + \cdots\,, \tag{20}$$

where $a = \frac{1}{2} + \sqrt{g + \frac{1}{4}}$ and K_1 is some positive constant. For $g = 2$, the corresponding solution is well-known [5]:

$$K(x) = \frac{x}{\sinh x}\,, \quad h(x) = \frac{\cosh x}{\sinh x} - \frac{1}{x}\,. \tag{21}$$

To study the case with general $g > 0$, we combine the above two equations into the following second-order equation for $L \equiv -\log K$ (here, $s \equiv \log x$):

$$\frac{d^2 L}{ds^2} + \partial_L U_{\text{eff}}(L) = \frac{dL}{ds}\,, \quad \left(U_{\text{eff}}(L) = -\frac{g}{2}(L + \frac{1}{2}e^{-2L})\right). \tag{22}$$

The problem is thus changed to that of a one-dimensional particle motion in the presence of an anti-damping force proportional to its velocity. The condition (20) now reads $L(s) = K_1 e^{\alpha s} + \cdots$ as $s \to -\infty$, while, from $K(\infty) = 0$, we must have $L(s) \to \infty$ as $s \to \infty$. Now think of $L(s)$ as the position of a "particle" that starts from the "point" $L = 0$ at "time" $s = -\infty$. Then, once we insists K_1 to be positive, the particle will always end up at $L = \infty$, because of the continuous increase of particle "energy" due to the $\frac{dL}{ds}$ term in (22) and the monotonically decreasing nature of U_{eff} for positive L. In fact, for sufficiently large s, we always have $L \sim Ce^S \to \infty$ with some positive constant C; then, using the first relation in (19), it follows that $0 < h(\infty) = C < \infty$ for generic values of $K_1 > 0$. On the other hand, a particular solution of (19) (with the boundary condition $h(\infty) = 1$ ignored), say $\bar{h}(x)$, generates a one-parameter family of solutions $h_\Lambda(x) = \Lambda \bar{h}(x/\Lambda)$ for all positive Λ. Using this freedom, we can always find the solution that satisfies the boundary condition $h(\infty) = 1$ as desired, whenever $g > 0$. (The last step is also tantamount to choosing a particular value of K_1.) This shows that there exist actual solutions saturating the BPS bound for the unit-charged cases.

What about multi-monopole solutions? In the usual $g = 2$ BPS model, static multi-monopole solutions are possible because the repulsive electromagnetic force is exactly bal-

anced against the attractive scalar interaction [8]. A tell-tale sign that this cancellation occurs may be found in the spatial components of the energy-momentum tensor T_{ij} that vanishes identically upon using the first-order BPS equation (9). Remarkably, exactly the same cancellation occurs for the present nonrenormalizable theories with arbitrary positive g also. This is because the structure of T_{ij} and that of the BPS equation remain unchanged once we replace G^a_{ij} and $(D_k\Phi)^a$ by $[G^a_{ij} - (\frac{g}{2}-1)\hat{\Phi}^a\mathcal{H}_{ij}]$ and $[(D_k\Phi)^a + (\frac{g}{2}-1)\phi(D_k\hat{\Phi})^a]$. This strongly suggests that there should exist static multi-monopole solutions to (17) also. In a way, this should have been anticipated by the following reason. As far as the long-range interaction between monopoles are concerned, the nonrenormalizable couplings introduced by the extra free parameter g do not seem to have significant effects. For our generalized, unit-charged BPS monopoles for example, one can show using (13) that the long-distance tails are given by

$$\hat{\Phi}^a G^a_{ij} \sim \pm\epsilon_{ijk}\frac{\hat{x}^k}{er^2}\,, \quad \Phi^a \sim \left[\phi(\infty) \pm \frac{1}{er}\right]\hat{x}^a\,, \tag{23}$$

which are entirely independent of the gyromagnetic ratio g.

The Lee-Weinberg model allows finite-energy dyon solutions also. But a rather surprising fact we might add here is that, if g is not equal to two, our model defined by the Lagrangian density (2) does *not* allow the BPS-type equations for dyons. (The case of BPS dyons for $g = 2$ is discussed in Ref. [9].) This is partially due to the nontrivial role assumed by the Gauss law constraint for dyon solutions.

To summarize, we have shown that a Dirac-string-free non-Abelian description is possible for the Lee-Weinberg $U(1)$ monopoles. When the Higgs sector is appropriately constrained, the theory admits the Bogomol'nyi-type bound which is saturated by purely magnetic configurations solving the generalized BPS equations. We have then explicitly demonstrated the existence of spherically symmetric one-monopole solutions satisfying the latter equations. Mass of these generalized BPS monopoles is independent of the gyromagnetic ratio g. Also conjectured is the existence of static multi-monopole solutions in this Bogomol'nyi limit.

It remains to see whether the generalized BPS monopoles discussed in this paper will

significantly enrich physics of solitons and lead to some new developments in mathematical physics. In any case, it is necessary to have a better understanding on the solution space of our generalized BPS equations. One may also look for analogous generalized BPS systems in the context of Yang-Mills-Higgs theories that are based on bigger gauge groups than $SO(3)$. Another interesting issue is whether these BPS systems can be understood as the bosonic sector of a suitable field theory possessing extended supersymmetry [10]. We hope to return to some of these outstanding problems in near future.

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